Corrected

QUANTIFICATION THEORY WITHOUT MULTIPLE OCCURRENCE-SETS OF VARIABLES*

R. B. Angell

Formation rules for quantification theory customarily sanction, as well-formed, formulae in which two or more quantifiers contain the same variable; most systems also permit a variable to occur free in some occurrences and bound in others. This paper shows the possibility of syntactical systems adequate for quantification theory in which wffs cannot contain the same variable in more than one quantifier or both bound and free. We construct a system of this sort, show it consistent and complete relative to Quine's quantification theory, and discuss briefly some consequences and possible extensions of the techniques used.

By an "occurrence-set of α in $\emptyset$", where $\emptyset$ is a quantificational formula and α is any variable which occurs in $\emptyset$, we mean either i) the set of all occurrences of α which are free in $\emptyset$, if such there be or ii) the set of all occurrences of α bound to some one quantifier in $\emptyset$ (including the occurrence in the quantifier). Thus the number of occurrence-sets of any α in $\emptyset$ will be equal to the number of occurrences of $\ulcorner(\alpha)\urcorner$ in $\emptyset$, plus 1 if α also occurs free in $\emptyset$. We are interested here in syntactical systems for quantification theory which would not contain any wffs which had multiple (more than one) occurrence-sets of any variable.

I

Quine's quantification theory, hereafter called Q1, permits variables to occur both free and in any number of quantifiers in a given wff. ([1], §14). Below we contrast the metalogical base of Q1 with that of Q2, a modification of Quine's theory which excludes multiple occurrence-sets of any variable in its wffs. Both are subject to the same metalogical standard interpretations, and both

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follow Quine's metalinguistic distinctions except for 1) explicit differences indicated by rules of formations and definitions below, 2) the introduction of a new symbol, '$^{\backprime}$', the accent, and 3) a broadened definition of 'tautology'.

Q1 PRIMITIVES

Parentheses:) (
Stroke: $\downarrow$
Epsilon(uninterpreted): ϵ
Variables (in "alphabetic order"):
$w\ x\ y\ z\ w'\ x'\ y'\ z'\ w''\ x''\ldots$
Quantifier: $\ulcorner(\alpha)\urcorner$ (where α is a variable)

Q2 PRIMITIVES

Same as in Q1

Q1 FORMATION RULES

Q1-F1. If α and β are variables, then $\ulcorner(\alpha \epsilon \beta)\urcorner$ is a formula.
Q1-F2. If ϕ is a formula, then $\ulcorner((\alpha)\phi)\urcorner$ is a formula.
Q1-F3. If ϕ and ψ are formulae, then $\ulcorner(\phi \downarrow \psi)\urcorner$ is a formula.
Q1-F4. Q1-formulae are all and only those obtainable by Q1-F1 through Q1-~~F4~~.F3

Q2 FORMATION RULES

Q2-F1. Same as in Q1
Q2-F2. If ϕ is a formula, then $\ulcorner((\alpha)\phi^{\backprime})\urcorner$ is a formula.
Q2-F3. If ϕ and ψ are formulae, then $\ulcorner(\phi \downarrow \psi)^{\backprime}\urcorner$ is a formula.
Q2-F4. Q2-formulae are all and only those obtainable by Q2-F1 through Q2-~~F4~~. F3.

Q1 DEFINITIONS

Q1-D1. $\ulcorner\sim\phi\urcorner$ for $\ulcorner(\phi\downarrow\phi)\urcorner$
Q1-D2. $\ulcorner(\phi.\psi)\urcorner$ for $\ulcorner(\sim\phi\downarrow\sim\psi)\urcorner$
Q1-D3. $\ulcorner(\phi \vee \psi)\urcorner$ for $\ulcorner\sim(\phi\downarrow\psi)\urcorner$
Q1-D4. $\ulcorner(\phi\supset\psi)\urcorner$ for $\ulcorner(\sim\phi\vee\psi)\urcorner$
Q1-D5. $\ulcorner(\phi\equiv\psi)\urcorner$ for $\ulcorner((\phi\supset\psi).(\psi\supset\phi))\urcorner$
Q1-D6. $\ulcorner(\phi.\psi.\chi)\urcorner$ for $\ulcorner((\phi.\psi).\chi)\urcorner$ etc.
Q1-D7. $\ulcorner(\phi\vee\psi\vee\chi)\urcorner$ for $\ulcorner((\phi\vee\psi)\vee\chi)\urcorner$ etc.
Q1-D8. $\ulcorner(\exists\alpha)\phi\urcorner$ for $\ulcorner\sim((\alpha)\sim\phi)\urcorner$

Q2 DEFINITIONS

Q2-D1. $\ulcorner\sim\phi\urcorner$ for $\ulcorner(\phi\downarrow\phi)^{\backprime}\urcorner$
Q2-D2. Same as in Q1.
Q2-D3. Same as in Q1.
Q2-D4. Same as in Q1.
Q2-D5. $\ulcorner(\phi\equiv\psi)\urcorner$ for $\ulcorner((\phi\supset\psi).(\psi\supset\phi)^{\backprime})\urcorner$
Q2-D6. Same as in Q1.
Q2-D7. Same as in Q1.
Q2-D8. Same as in Q1.

Q1 INITIAL METATHEOREMS

Q1-*100. If ϕ is tautologous, $\vdash\phi$.
Q1-*101. $\vdash\ulcorner((\alpha)(\phi\supset\psi))\supset.((\alpha)\phi)\supset((\alpha)\psi)\urcorner$
Q1-*102. If α is not free in ϕ, $\vdash\ulcorner\phi\supset(\alpha)\phi\urcorner$
Q1-*103. If ϕ' is like ϕ except for containing free occurrences of α' wherever ϕ contains free occurrences of α, $\vdash\ulcorner(\alpha)\phi\supset\phi'\urcorner$
Q1-*104. If $\ulcorner(\phi\supset\psi)\urcorner$ and ϕ are theorems, so is ψ.

Q2 INITIAL METATHEOREMS

Q2-*100. If ϕ is tautologous, $\vdash\phi$.
Q2-*101. $\vdash\ulcorner((\alpha)(\phi\supset\psi))\supset.((\alpha)\phi)^{\backprime}\supset((\alpha)\psi)^{\backprime}\urcorner$
Q2-*102. $\vdash\ulcorner\phi\supset(\alpha)\phi^{\backprime}\urcorner$
Q2-*103. $\vdash\ulcorner(\alpha)\phi\supset\phi^{\backprime}\urcorner$
Q2-*104. Same as in Q1.

The only deviation in Q1 from Quine is the harmless one, inQ1-F2, of enclosing primitive quantifications in parentheses; this makes possible in Q2 a distinction between an accent applied to a matrix (as in Q2-*102) and an accent applied to a whole qumatification (as in Q2-*101). It is ignored elswhere. We shall now explain the differences between Q1 and Q2 and the reasons for them.

II

Most of the differences between Q1 and Q2 are conveyed by the metalogical use of accents. The accent is interpreted as follows:

> $\ulcorner(\ldots\phi^{\circ}\ldots)\urcorner$ means " any expression, χ, just like $\ulcorner(\ldots\phi\ldots)\urcorner$ except that if ϕ contains any α which has more than one occurrence-set in $\ulcorner(\ldots\phi\ldots)\urcorner$, then the occurrence-sets of α in ϕ are re-lettered, beginning with bound sets, so that no variables in ϕ have more than one occurrence-set in χ."

Accents occur only in quasi-quotations and may apply to parenthesized components as well as to metalogical variables for formulae.

<u>Q2-Formation Rules.</u> Since Q2 Formation Rules differ from Q1 Formation Rules only in the occurrence of accents in $\ulcorner((\alpha)\phi^{\circ})\urcorner$ in Q2-F2 and in $\ulcorner(\phi\downarrow\psi)^{\circ}\urcorner$ in Q2-F3,

**1 All expressions which are Q2-formulae are also Q1-formulae.

For, the only sort of difference introduced by accents occurs by re-lettering, which preserves formula-hood in Q1.

If ϕ is a Q2-formula, the application of Q2-F2 to ϕ yields the same result as Q1-F2 provided ϕ either contains only free occurrences of α or contains no occurrences of α. If and only if α occurs bound in ϕ, creating two or more occurrence-sets of α in $\ulcorner((\alpha)\phi)\urcorner$, does the accent require that those bound occurrence-sets be re-lettered to get a formula $\ulcorner((\alpha)\phi^{\circ})\urcorner$. Again, in Q2-F3, given two Q2-formulae ϕ and ψ, the application of Q2-F3 to get $\ulcorner(\phi\downarrow\psi)^{\circ}\urcorner$ yields the same result as Q1-F3 as long as no variable occurs in both ϕ and ψ and is bound in one of them. If and only if the result $\ulcorner(\phi\downarrow\psi)\urcorner$ without the accent would involve multiple occurrence-sets, does the accent require a re-lettering. Since in both rules the accent re-quires re-lettering if and only if multiple occurrence-sets would otherwise occur,

> **2 Q2-formulae are all and only those Q1-formulae without multiple occurrence-sets of any variable.

And since, when re-eltering occurs, bound occurrence-sets are always re-lettered first, free variables in $\ulcorner((\alpha)\phi^{\circ})\urcorner$ or in $\ulcorner(\phi\downarrow\psi)^{\circ}\urcorner$ will not be re-lettered since, when reached, they will be the sole occurrence-set of the variable involved. It follows from this, and Quine's definition of alphabetic variants (cf. §21[1]),

**3 Each application of Q2-F2 or Q2-F3 to a given Q2 formula or pair of formulae, produces an expression which is either the same as or an alphabetic variant of the result of applying Q1-F2 or Q1-F3 to the same formula.

And from this it can be shown that

**4 Any sequence of applications of rules Q2-F1, Q2-F2 and Q2-F3 will result either in the same expression or in an alphabetic variant of the expression produced by a parallel sequence of applications of rules Q1-F1, Q1-F2 and Q1-F3.

For, the construction of formulae begins in both Q1 and Q2 with the same rule for atomic formulae, and alphabetic variance will be preserved through each pair of steps of the parallel sequences.

Q2-Abbreviations. In Q2, as in Q1, abbreviations are shorthand and remarks about them really apply to corresponding expressions in unabbreviated notation. Nevertheless the metalogical formulations above make use of abbreviations, and because of this, changes in D1 and D5 are necessary to preserve the completeness of the system outlined by the Q2 metatheorems. If, for example, $\ulcorner(\phi\downarrow\phi)\urcorner$ were left as the the definiens of $\ulcorner\sim\phi\urcorner$ in Q2, the only formulae of Q2 which could be abbreviated by negations signs would be quantifier-free formulae, since other expressions of that form would have multiple occurrence-sets and thus not be Q2-formulae at all. Since negation enters in to each of D2 through D8, no abbreviation could abbreviate any expression containing quantifiers and the existential quantifer would be meaningless since none of its instances, requiring two occurrences of $\ulcorner(\alpha)\urcorner$, could be Q2-formulae. Thus $\ulcorner\sim\phi\urcorner$ is defined in Q2 as $\ulcorner(\phi\downarrow\phi^{\circ})\urcorner$ which will replace multiple occurrence-sets by alphabetic variants and permit the negation sign in Q2 to abbreviate all the correlates (by **4) in Q2 of what it can abbreviate in Q1. In like manner, and for somewhat similar reasons, we define $\ulcorner(\phi\equiv\psi)\urcorner$ in Q2 as $\ulcorner((\phi\supset\psi).(\psi\supset\phi)^{\circ})\urcorner$ in Q2-D5 so as to include in Q2 alphabetic variants of Q1 biconditionals in which at least one quantifier occurs in ϕ or ψ even though they have no variable in common. Further changes are not necessary in Q2 definitions since none of the other six contain more than one occurrence of ϕ, ψ or $\ulcorner(\alpha)\urcorner$. The changes above, together with **4 are sufficient to establish that

**5 Every abbreviated expression of Q2 is either reducible to the same primitive notation or to an alphabetic variant of the primitive notation of the corresponding abbreviated expression in Q1.

The reduction of abbreviations to primitive notation in Q2 differs from that process in Q1 in that the presence of quantifiers in the definiendum of a negation or biconditional will require the introduction of new variables in the definiens in order to avoid multiple occurrence sets. This requirement, however, has ample precedent in Quine's D9, D10 and D12 (abstraction and identity), and poses no problem.

Q2-Metatheorems. The completeness and consistency of Q2 will depend upon showing, for each metatheorem Q2-*100 through Q2-*103, that a) each axiom under the given metatheorem is either the same as or an alphabetic variant of a Q1-axiom under the corresponding Q1 metatheorem, (hence axioms of Q2 are always theorems of Q1) and b) the theorems of Q1 can be put into 1-1 correspondence with a class of theorems of Q2.

With respect to Q2-*100, it is necessary to broaden the concept of tautology if Q2 is to be complete with respect to Q1. Truth-functional tautology in Q1 depends entirely upon having repetitions of one or more components in the tautologous formula. But if the repeated components contained quantifiers, as they often do, then the result would be a non-formula in Q2 due to multiple occurrence-sets, and thus many tautologies of Q1 would be without counter-parts in Q2 and Q2 would be incomplete with respect to Q1. This situation is alleviated by re-defining tautology by the addition of the stipulation that in any truth-table all components which are alphabetic variants of each other must be assigned identical truth-values in each possible case. Thus, for example, the truth-table for '((x)xex ⊃ (y)yey)' becomes

(x)xex	⊃	(y)yey
T	T	T
F	T	F

rather than

(x)xex	⊃	(y)yey
T	T	T
F	T	T
T	F	F
F	T	F

This broadening of the concept of tautology is unobjectionable since alphabetic variants are effectively distinguishable within any given formula, and as Quine says,

> Variables, as remarked in (§12), serve merely for cross-reference to various positions of quantification. The particular choice of letters which happens to be made, in constructing a statement, is immaterial to the meaning so long as the system of cross-feferences remains the same...[Alphabetic variants] differ from one another only in so accidental detail of notation. If instead of using variables we were to indicate the cross-references by the method of curved lines (cf.§12), dáfferences of this sort would drop out altogether. ([1],pp 109-110)

With the redefinition of tautology the class of Q2-tautologies is broadeded to include $\ulcorner(\phi \supset \phi^{\circ})\urcorner$ and $\ulcorner(\phi \equiv \phi^{\circ})\urcorner$, etc., since accented expressions will always be either the same as, or alphabetic variants of the unaccented expressions which would be Q1 tautologies. It follows from this and **4 that,

> **6 Each Q2 tautology, and thus each axiom in Q2 by Q2-*100, will be either the same as, or an alphabetic variant of, some axiom of Q1-*100.

Incidently this change in definition of tautology renders it unnecessary to have analogues in Q2 of the Q1 metatheoremm on alphabetic variance,*170 and *171.

The metatheorem Q1*101 would be meaningless in Q2, as it stands, since the three occurrences of $\ulcorner(\alpha)\urcorner$ call for three occurrence-sets of the same variable and Q2 contains no formulae of that sort. In Q2-*101, accents are placed on the scopes of the second and third occurrences of $\ulcorner(\alpha)\urcorner$ and it follows that

> **7 Each Q2 axiom by Q2-*101 will be an alphabetic variant of some axiom by Q1-*101.

For,when $\ulcorner(\alpha)(\phi \supset \psi)\urcorner$ does not have multiple occurrence-sets, the accents in $\ulcorner((\alpha)(\phi \supset \psi) \supset .((\alpha)\phi)^{\circ} \supset ((\alpha)\psi)^{\circ})\urcorner$ merely re-letter the second and third occurrence-sets of α, producing an alphabetic variant of the corresponding Q1-axiom; and if $\ulcorner((\alpha)(\phi \supset \psi))\urcorner$ does contain multiple occurrence-sets, then it can not be a Q2 formula or a component in a Q2 formula, and hence this case will not pertain.

With respect to Q2-*102, it is unnecessary to include the qualifying phrase which occurs in Q1, "If α is not free in ϕ," since if α did occur free in ϕ in $\ulcorner(\phi \supset (\alpha)\phi)\urcorner$ there would be two occurrence-sets of α , one in ϕ and one in the quantifier, and the result would not be a Q2 formula or axiom. Adding an accent to the second occurrence of $\emptyset$, as in $\ulcorner(\emptyset \supset (\alpha)\emptyset')\urcorner$, however, makes it possible to utilize $\emptyset$s with any number of distinct quantifiers and to establish,

> **8 Each Q2 axiom by Q2-*102 will be either the same as, or an alphabetic variant of, some axiom in Q1 by Q1-*102.

For, no Q2 axiom will have α free in $\emptyset$, thus that condition is always satisfied. If $\emptyset$ contains no quantifier, the Q2 axiom by Q2-*102 will be the same as the Q1 axiom by Q1-*102. If $\emptyset$ contains one or more quantifiers without having multiple occurrence-sets, then $\emptyset$ is a Q2 formula as well as a Q1 formula, but must be re-lettered in all bound occurrences according to the accent in Q2-*102, and thus the axiom by Q2-*102 will be an alphabetic variant of the axiom by Q1-*102. If $\emptyset$ contains quantifiers with multiple occurrence-sets, it will not be a Q2 formula, no Q2 axiom will result, and thus this case does not pertain.

In Q2-*103 it is unnecessary to prefix the qualification, "If $\emptyset'$ is like $\emptyset$ except for containing free occurrences of α' wherever $\emptyset$ contains free occurrences of α , then ...", since the accent assures that any occurrence of α in $\emptyset$ will be re-lettered in $\emptyset'$; indeed, the case of $\alpha = \alpha'$ can not occur in Q2-*103, since it would entail two occurrence-sets of the same variable. It still holds, however, that

> **9 Each Q2 axiom by Q2-*103 will be either the same as, or an alphabetic variant of, some axiom of Q1 by Q1-*103.

For, if $\emptyset$ contains no quantifiers and if either $\alpha \neq \alpha'$ or $\emptyset$ does not contain α , then $\emptyset$ is a Q2 formula, and the resulting Q2 axiom by Q2-*103 is the same as that which would result by Q1-*103. If $\emptyset$ does contain quantifiers - other than $\ulcorner(\alpha)\urcorner$ - but no multiple occurrence-sets, then $\emptyset$ must be re-lettered under the accent and the result of Q2-*103 will be an alphabetic variant of the result of Q1-*103 with $\alpha \neq \alpha'$. If $\emptyset$ contains $\ulcorner(\alpha)\urcorner$ and/or multiple occurrence-sets of ~~quantifiers~~ variables, it will not pertain to **9 since it would fail to be a Q2 formula.

The Q2-metatheorems, Q2-*100 through Q2-*103, designate, in effect, four infinite classes of axioms of Q2. Q2-*104, which is the same as in Q1, is the sole primitive rule of inference, modus ponens, by means of which additional theorems are generated from axioms.

III

From the preceding considerations it is easy to establish the consistency of Q2 with respect to Q1. For, every Q2 axiom will be a Q1 theorem, since 1) by **6, **7, **8 and **9, each Q2 axiom by Q2-*100, Q2-*101, Q2-*102 or Q2-*103, will be either the same as or an alphabetic variant of some Q1 axiom by Q1-*100, Q1-*101, Q1-*102 or Q1-*103 respectively, and 2) the groups of axioms above exhaust all the axioms of Q2, and 3) where any Q2 axiom is an alphabetic variant of a Q1 axiom, it is easily proved to be a theorem of Q1 by Q1-*171 (on the equivalence of alphbatic variants), and Q1-*124. But if Every Q2 axiom is a Q1 theorem, and the sole rule of inference, modus ponens, is the same in both systems, then all Q2 theorems - i.e., the ponentials of Q2 axioms of quantification, together with the ponentials of this totality, and so on - will be a sub-class (by the same definition of theorems) of the class of Q1 theorems. If the class of Q1 theorems is consistent, therefore, the class of Q2 theorems must be consistent also.

IV

The completeness of Q2 with respect to Q1 is a bit more difficult. It is proven by showing that the theorems of Q1 can be put into one-to-one correspondence with a sub-class of the theorems of Q2. We begin by putting the formulae of Q1 into 1-1 correspondence with a sub-class of the formulae of Q2 according to the following rule:

Rule I: If $\emptyset i$ is any Q1-formula, $\emptyset i'$ is formed from $\emptyset i$ by replacing each occurrence of an (i,k)th variable in $\emptyset i$ by an occurrence of the $(2^k \cdot (2i-1))$th variable,

where " an (i,k)th variable in $\emptyset$" means " an occurrence of the i^{th} variable according to alphabetic order in the list of primitives, in its k^{th} occurrence-

set in ϕ" and the k^{th} occurrence-set of α in ϕ is either the occurrence-set bound to the k^{th} occurrence of ⌜(α)⌝ (reading left to right) if α does not occur free in ϕ, or else the occurrence-set bound to the $(k-1)^{th}$ occurrence of ⌜(α)⌝ (reading left to right) with the 1st occurrence-set ~~be~~ being that of free α.
Rule I puts the class of Q1 formulae into 1-1 correspondence with a part of itself. But that part, or sub-class, of Q1 formulae is one in which no member formula has multiple occurrence-sets of any variable, since each k^{th} occurrence-set in ϕi is given a different variable in ϕi'. Thus the class of Q1 formulae is put into 1-1 correspondence with a sub-class of Q2 formulae.

Now a <u>theorem</u> in either Q1 or Q2 is simply the last formula in a formal deduction. And a <u>formal deduction</u> in either Q1 or Q2 is simply a list of formulae each of which is either an axiom of the system, or a ponential of (a result of Q1-*104 or Q2-*104, <u>modus ponens</u>) two ealier formulae on the list (Cf. [1],p.319). We next put the class of Q1 formal deductions into 1-1 correspondence with a certain class of lists of Q2 formulae by

<u>Rule II</u>: Given any Q1 formal deduction, Q1FDi, a list, FDi' is formed by replacing each ϕ in Q1FDi by ϕi' according to Rule I.

Since each ϕi' is a Q2-formula, each list FDi' will be a list of Q2 formulae, and these lists will stand in 1-1 correspondence, formula for formula and list by list, with the Q1 formal deductions.

We next prove that

**10 If any formula ϕi in a Q1 formal deduction Q1FDi is a Q1 axiom, then the corresponding formula Qi' in the corresponding list of Q2 formulae, FDi', is a Q2 axiom.

For every formula ϕj' which corresponds to ~~an~~ a Q1 axiom, ϕj, by Rule I, will be an alphabetic variant without multiple occurrence-sets of that Q1 axiom. But if any formula ψ is an alphabetic variant without multiple occurrence-sets of a Q1 axiom, it must be a Q2 axiom. For the formula ψ will have the same form and structure, apart from the lettering of variables, as the Q1 axiom and the only difference between Q1 axioms and their Q2 correlatives is in the re-lettering of variables when this occurs to avoid multiple occurrence-sets. Since no restriction is introduced, by the accent, on <u>which</u> alphabetic variants without multiple occurrence-sets come under the metatheorems Q2-#100 to Q2-#103, it follows that all such alphabetic variants fall under it equally, including those formed by Rule I.

Unfortunately, we can not show that if ϕj is a ponential of two earlier formulae in any Q1-formal deduction, then $\phi j'$ is a ponential of two earlier formulae in ~~that~~ the corresponding FDi' list of Q2 formulae. Compare the examples:

$$\begin{array}{ll} Q1FD_1: & (w)(wew \supset wew), (w)(wew \supset wew) \supset (w)wew \supset (w)wew, \quad (w)wew \supset (w)wew \\ FD_1': & (x)(xex \supset xex), (x)(xex \supset xex) \supset (x')x'ex' \supset (x'')x''ex'', \quad (x)xex \supset (x')x'ex' \end{array}$$

The formal Q1 deduction, $Q1FD_1$, has the form: ϕi, $\phi i \supset \phi j$, ϕj. But its correlate by Rule II, FD_1', has the form: $\phi i'$, $(\phi i \supset \phi j)'$, $\phi j'$ where $(\phi i \supset \phi j)'$ is <u>not</u> the same as $\phi i' \supset \phi j'$ but has the form $\phi i' \supset \Psi$ where $\phi j' \neq \Psi$. Thus while ϕj is a ponential in $Q1FD_1$, $\phi j'$ is not a ponential in FD_1'.

However, it can still be proven that

> *#11 If ϕj is a ponential in a Q1 formal deduction, Q1FDi, then $\phi j'$ will be a ponential in a uniquely determined Q2 formal deduction, Q2FDi'.

For wherever ϕj is a ponential in some Q1FDi, we can replace $\phi j'$ in FDi', by the sequence $\Psi, \Psi \supset X, X, X \supset \phi j', \phi j'$, where Ψ and X are determined by the definitions,

$$\begin{array}{llll} \phi i' \supset \Psi & = df & (\phi i \supset \phi j)' & \text{(according to Rule I)} \\ \Psi \supset X & = df & (\Psi \supset \Psi)' & \text{(according to Rule I)} \end{array}$$

and the result will be a uniquely determined Q2 formal deduction, in 1-1 correspondence with just one Q1 formal deduction (though not having all formulae in the lists in 1-1 correspondence). The introduced hypotheticals are axioms of Q2 since their components are alphabetic variants, and the other formulae are true Q2 ponentials of earlier formulae on the list. Since the the terminal formulae in each list will remain the same, and Q2 formal deductions of this sort can be produced for every Q1 formal deduction, we have shown that the Q1 theorems can be put into 1-1 correspondence with a sub-class of Q2 theorems and thus Q2 is complete with respect to Q1.

VI

Differences in the initial metatheorems of Q2 and Q1 entail, of course, that there would have to be many differences in the development of subsequent metatheorems and proofs of metatheorems in the two systems. This does not affect the soundness of the system Q2, but it leaves open the question of whether Q2 could be developed as perspicuously as Quine's development of Q1. We have seen certain economies, on the surface at least, in eliminating qualifying phrases about occurrences of free variables, and in dispensing with metatheorems *170 and *171. But whether there would be a net economy or not can not be settled unless such a reconstruction is carried out.

Two points can be made. It would be possible to eliminate the accent entirely from our metalogical formulation, and without further changes, if it were undesstood that two occurrences, or more, of 'ϕ', 'ψ', '(α)', etc., in a quasi-quotation were always to be treated so that quantifiers would be re-lettered to avoid multiple occurrence-sets; the logician would have to develop a knack, or method, for identifying similar formulae in terms of patterns of cross-reference regardless of the variables used and making it a practice to use new variables whenever a quantifier was introduced. It might not be so difficult as might appear. In this paper we have followed the practice of assuming, as is standard, that two occurrences of ϕ, $\ulcorner(\alpha)\urcorner$, etc., in a quasi-quotation must designate identical expressions in each occurrence. This facilitated the comparison with Q1, but is not essential in metalogic.

Finally, it seems clear that since the results of this paper rest entirely on principles concerning alphabetic variance which are accepted in all formulations of qunstification theory, there should be no great diffculty in transferring the devices and techniques and proofs used to justify our results in connection with Q1, to similar endeavors in connection with other standard formulations of quantification theory.

Bibliography:

[1] Quine, W.V.O. *Mathematical Logic*, Harvard, 1958.